Key

Math 1

Lesson 2-2 Arithmetic Sequences

Name____________________

Date________

Learning goals:

- *I can convert a sequence into a recursive or explicit formula.*
- *I can use a formula to find missing terms in a sequence.*
- *I can determine the common difference/ratio from a sequence.*
- *I can construct a linear or exponential function from an arithmetic sequence, table of values or verbal description.*

A sequence is an **ordered list of numbers**. The sequence is named with a letter and a subscript, for example a_n. The letter is used to name the sequence, and the subscript ("n") refers to the position of a number in the sequence. For instance, a_4 would refer to the 4th term in sequence a. a_n would be the n^{th} term in the sequence. Suppose a_n was the 8[th] term in a sequence. a_{n-1} would be the ____________ term.

Given the sequence $\{0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ ...\}$

Find the following terms: $a_4 =$ 2 ; $a_1 =$ 0 ; $a_9 =$ 21 ;

Formulas for Sequence

Some sequences have formulas that generate them. Formulas can be either explicit or recursive. An **explicit formula** is a formula that allows direct computation of any term in the sequence. A **recursive formula** requires the computation of all previous terms in order to find the value of term a_n.

Consider the sequence $\{2,\ 4,\ 6,\ 8,\ 10,\ 12,\ 14,\ 16,\ ...\}$

The *explicit formula* for this sequence is $a_n = 2n$

The *recursive formula* for this sequence is $\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 2 \end{cases}$

Note that for the recursive formula, you must know the first term, then you define the rest of the terms in relation to the first term.

Write out the first four terms of the following sequences:

$g_n = 2n - 1$ $\quad 2(1)-1 =$ 1 , $2(2)-1 =$ 3 , 5 , 7

$g_1 \quad g_2 \quad g_3 \quad g_4$

$k_i = i^2 - 1$ $\quad (1)^2-1 =$ 0 , 3 , 8 , 15

$k_1 \quad k_2 \quad k_3 \quad k_4$

$$\begin{cases} q_1 = 3 \\ q_n = 2 \cdot q_{n-1} \end{cases}$$

3, 6, 12, 24

q_1 q_2 q_3 q_4

$$\begin{cases} b_1 = 4 \\ b_n = 2 \cdot (b_{n-1}) - 3 \end{cases}$$

4, 5, 7, 11

b_1 b_2 b_3 b_4

$2 \cdot 4 - 3 = 5$ $2 \cdot 5 - 3 = 5$

III. **Arithmetic Sequences**

An ***arithmetic sequence*** is a sequence in which the difference between two consecutive terms is the same. The *common difference* is found by subtracting any term from its succeeding term. The n^{th} *term* (a_n) of an arithmetic sequence with first term a_1 and the common difference is d is given by the following formula: $a_n = a_1 + (n-1)d$

A. Name the first four terms of each **arithmetic sequence** defined below. An example is given.

Example: $a_1 = 2,\ d = 3$ → 2, 5, 8, 11

1. $a_1 = 4,\ d = 3$

4, 7, 10, 13

2. $a_1 = 7,\ d = 5$

7, 12, 17, 22

3. $a_1 = -\frac{4}{5},\ d = 1$ -0.8, 0.2, 1.2, 2.2

$-\frac{4}{5}$, $\frac{1}{5}$, $\frac{6}{5}$, $\frac{11}{5}$

B. Name the next four terms of each of the following **arithmetic sequences**:

Example: 5, 9, 13

4

17, 21, 25, 29

1. 21, 15, 9

-6

3, -3, -9, -15

2. 11, 14, 17

3

20, 23, 26, 29

3. 9.9, 13.7, 17.5

3.8

21.3, 25.1, 28.9, 32.7

C. Write out the first four terms of the following arithmetic sequences; then write an explicit and recursive formula.

	Explicit	Recursive
Example: $a_1 = 7,\ d = 3$ 7, 10, 13, 16	$a_n = 7 + (n-1)3$	$\begin{cases} a_1 = 7 \\ a_n = a_{n-1} + 3 \end{cases}$
1. $a_1 = -1,\ d = 10$ -1, 9, 19, 29	$a_n = -1 + (n-1)10$	$\begin{cases} a_1 = -1 \\ a_n = a_{n-1} + 10 \end{cases}$
2. $a_1 = 2,\ d = \frac{1}{2}$ 2, 2.5, 3, 3.5	$a_n = 2 + (n-1)0.5$	$\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 0.5 \end{cases}$
3. $a_1 = 27,\ d = -16$ 27, 11, -5, -21	$a_n = 27 + (n-1)(-21)$ or $a_n = 27 - 21(n-1)$	$\begin{cases} a_1 = 27 \\ a_n = a_{n-1} - 16 \end{cases}$

D. Find the indicated term in each **arithmetic sequence**:

Example: a_{12} for -17, -13, -9, . . .

$+4$

$a_{12} = -17 + (12-1)(4)$

$= -17 + 11 \cdot 4$

$= -17 + 44$

$a_{12} = \boxed{27}$

1. a_{21} for 10, 7, 4

-3

$a_{21} = 10 + (21-1)(-3)$

$\boxed{a_{21} = -20}$

2. a_{10} for 8, 3, -2

-5

$a_{10} = 8 + (10-1)(-5)$

$\boxed{a_{10} = -37}$

3. a_{12} for 3, 4.1, 5.2, 6.3

$a_{12} = 3 + (12-1)1.1$

$\boxed{a_{12} = 15.1}$

E. Answer the following.

Example: Which term of $\overset{7}{\frown}$ -2, 5, 12, . . . is 124?

$124 = -2 + (n-1)\cdot 7$

$\frac{126}{7} = \frac{(n-1)7}{7}$

$18 = n-1$

$\boxed{19 = n} \rightarrow 19^{th}$ term

1. Which term of -3, 2, 7, . . . is 142?

$142 = -3 + (n-1)5$

$145 = (n-1)5$

$29 = n-1$

$\boxed{30^{th} \text{ term}}$

2. Which term of 7, 2, -3, . . . is -28?

$-28 = 7 + (n-1)(-5)$

$-35 = (n-1)(-5)$

$7 = n-1$

$\boxed{8^{th} \text{ term}}$

F. Find the missing terms of the following arithmetic sequences:

Example: 55, 70, 85, 100, 115

$55 + d(4) = 115$

$4d = 60$

$d = 15$

1. -10, -8.8, -7.6, -6.4, -5.2, -4

$-10 + d(5) = -4$

$5d = 6$

$d = \frac{6}{5} = 1.2$

2. 2, 5, 8, 11, 14, 17, 20

$2 + d(6) = 20$

$6d = 18$

$d = 3$

G. Answer the following:

1. Bill is spending one day raking leaves for his neighbor. His neighbor will pay him \$9.50 an hour, plus \$7 for lunch money which will be paid at the start of the day.

   a. List how much money would be paid to Bill for each of the first four hours.

   16.50, 26, 35.5, 45
   1 hr, 2 hr, 3 hr, 4 hr

   b. All of our previous problems started with a_1, what would a_1 represent in this situation?

   How much Bill gets paid the first hour.
   ★★★ $a_0 \rightarrow$ How much he gets to start (zero hours of work)

   c. Write an explicit formula that models Bill's pay.

   $B = 7 + 9.50t$

   B = Bill's pay
   t = # of hours worked

   d. Write a recursive formula that models Bill's pay.

   $$\begin{cases} B_0 = 7 \\ B_n = B_{n-1} + 9.5 \end{cases}$$

   e. Which formula would be best to use to answer the following question: How much money will Bill get paid if he works for 12 hours? Solve it.

   Explicit!

   $B = 7 + 9.50(12)$
   $\boxed{= \$121}$

   f. How many hours did Bill rake leaves for if he got paid \$73.50?

   $73.50 = 7 + 9.50t$

   $66.50 = 9.50t$

   $\boxed{t = 7 \text{ hours worked}}$